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Short Communication

The A_T -{Transmuted - X} Family of Distributions

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Abstract

The Exponential Transmuted Exponential distribution (ETE) appeared in and in this paper we present a new generalization of the ETE distribution based on an application of the Ampadu-G family of distributions that appeared in [1,2]. We also show the new distribution is a good fit to some real-life data, indicating practical significance. As a further development, we propose a new class of distributions based on the structure of the weight function introduced in [3].

Keywords: A_T - X(W) family of distributions; Exponential Distribution; Transmuted Distribution

Preliminaries

At first we recall the following definitions

Definition 1.1.

Let $\lambda > 0$, $\xi > 0$ be a parameter vector all of whose entries are positive, and $x \in$ R [2]. A random variable X will be said to follow the Ampadu-G family of distributions if the CDF is given by

$$F(x;\lambda,\xi) = \frac{1-e^{-\lambda G(x;\xi)^2}}{1-e^{-\lambda}}$$

And the PDF is given by

$$f(x;\lambda,\xi) = \frac{2\lambda g(x;\xi)G(x;\xi)e^{-\lambda G(x;\xi)^2}}{1-e^{-\lambda}}$$

Where the baseline distribution has CDF G(x; ξ) and PDF g(x; ξ)

Definition 1.2.

Assume the random variable T with support [0,∞) has CDF G(t; ξ) and PDF g(t; ξ) [2]. We say a random variable S is A_T - X(W) distributed of type II if the CDF can be expressed as either one of the following integrals

$$\int_0^{-log(1-F(x;\omega))} \frac{2\lambda g(t;\xi)G(t;\xi)e^{-\lambda G(t;\xi)^2}}{1-e^{-\lambda}}dt = \frac{1-e^{-\lambda G(-log(1-F(x;\omega));\xi)^2}}{1-e^{-\lambda}}$$

or

$$\int_0^{\frac{F(x;\omega)}{1-F(x;\omega)}} \frac{2\lambda g(t;\xi)G(t;\xi)e^{-\lambda G(t;\xi)^2}}{1-e^{-\lambda}}dt = \frac{1-e^{-\lambda G\left(\frac{F(x;\omega)}{1-F(x;\omega)};\xi\right)^2}}{1-e^{-\lambda}}$$

Where λ, ξ > 0, and the random variable X with parameter vector ω has CDF F(x; ω) and PDF f(x; ω)

Application

As an application we consider the data on patients with breast cancer [1]. We assume the random variable T is exponentially distributed. So that the CDF of T is given by

$$G(t;d) = 1 - e^{-dt}$$

For t, $d > 0$ and the PDF is given by

$$g(t;d) = de^{-d}t$$

For t, $d > 0$. Now we recall the following, for some baseline distribution $K(x)$, the transmuted family of distributions has CDF given by

$$(1 + b)K(x) - bK^2(x)$$

Where $b \in [-1, 1]$. Now we assume X is transmuted exponentially distributed, so that if

$$K(x) := 1 - e^{-ax}$$

For x, $a > 0$, then the CDF of X is given by

$$F(x;a,b) = (b + 1)(1 - e^{-ax}) - b(1 - e^{-ax})^2$$

And the PDF is given by

$$f(x;a,b) = a(b + 1)e^{-ax} - 2abe^{-ax}(1 - e^{-ax})$$

Now from the first integral in 3.2 Definition, we have the following

Theorem 2.1.

The *CDF* of the $A_{\{Exponential\}}$-{Transmuted Exponential} distribution is given by

$$S(x;a,b,c,d) = \frac{1-\exp\left(-c\left(1-\left(b\left(1-e^{-ax}\right)^2-(b+1)\left(1-e^{-ax}\right)+1\right)^d\right)^2\right)}{1-e^{-c}}$$

Where x, a, c, d > 0 and b ϵ [-1, 1]

Remark 2.2.

If a random variable Q has CDF given by the above theorem, write

$$Q \sim AETE(a, b, c, d)$$

The PDF of the AETE distribution can be obtained by differentiating the CDF

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The AETE distribution is seen to be a good fit to real life data as shown in the figure 1.

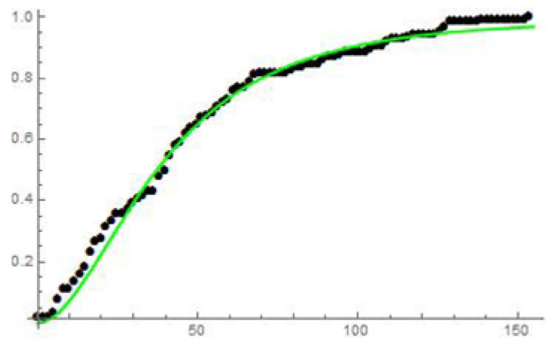

Figure 1: The CDF of AETE(0.0133756, 0.216229, 3.93753, 0.910041) fitted to the empirical distribution of the data on patients with breast cancer [1].

Further Developments

Inspired by the structure of the weight function introduced in, we ask the reader to investigate some properties and applications of a so-called $A_{\{New\ T\}}$ - $X(W)$ family of distributions of type II [3]. We leave the reader with the following.

Definition 3.1.

Assume the random variable T with support [0,∞) has CDF G(t; ξ) and PDF g(t; ξ). We say a random variable S_{New} is $A_{\{New\ T\}}$ - X(W) distributed of type II if the CDF can be expressed as the following integral

$$\int_0^{\frac{-log(1-F(x;\omega))}{1-F(x;\omega)}} \frac{2\lambda g(t;\xi)G(t;\xi)e^{-\lambda G(t;\xi)^2}}{1-e^{-\lambda}}dt = \frac{1-e^{-\lambda G\left(\frac{-log(1-F(x;\omega))}{1-F(x;\omega)};\xi\right)^2}}{1-e^{-\lambda}}$$

Where λ, ξ > 0, and the random variable X with parameter vector ω has CDF F(x; ω) and PDF f(x; ω)

Concluding Remarks

Our hope is that the new class of distributions presented in this paper will find application in cancer modeling and forecasting

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